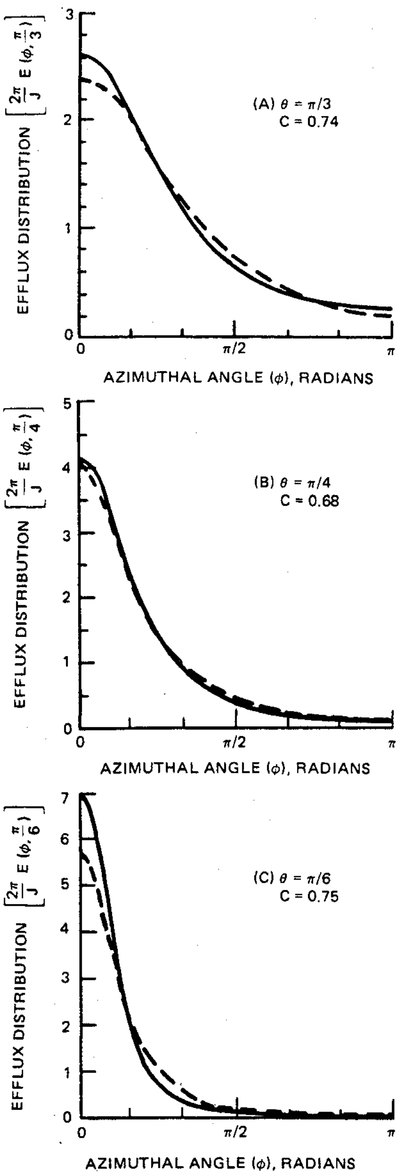

Fig. 3 Azimuthal distribution of efflux momentum;--- Taylor[5] experiment (C is inferred discharge coefficient), ——Schach[6] model [Eq. (6)].

Using Eqs. (6) and (8) in conjunction with Eq. (9) yields

$$\frac{2\pi}{J} I(\chi,\theta) = \frac{\sin^3\theta}{(1-\cos^2\theta\sin^2\chi)^{3/2}} \qquad (10)$$

which shows that the normalized distribution is symmetric about $\chi = \pi/2$ for all values of θ. This, too, is in agreement with Taylor's observations.[1,5] In addition, it is apparent that the peak magnitude of the normalized internal force distribution is given by $(2\pi/J)I(\pi/2,\theta) = 1$ and is independent of incidence angle. Table 2 demonstrates that Taylor's[5] data are again consistent with the Schach model except, perhaps, at $\theta = \pi/6$.

Conclusions

The model of Schach for round jet impingement yields a useful expression [Eq. (6)] for the azimuthal distribution of momentum efflux that compares well with data[5,6] over the range $\pi/6 \leq \theta \leq \pi/2$. Moreover, the model satisfies subtle characteristics of the flowfield uncovered by Taylor. In particular, the lateral to normal reaction forces on the symmetric half-jet are in the ratio of $2/\pi$, independent of jet incidence angle. Also, the distribution of internal forces parallel to the surface are in accord with his observations.

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Turbulent Boundary-Layer Flow over Re-entry Bodies Including Roughness Effects

P. R. Gogineni*
Lockheed Missiles and Space Company,
Sunnyvale, California

Nomenclature

C_f = skin-friction coefficient
k = roughness height
Pr = Prandtl number
R = radius
Re = Reynolds number
S = surface distance
St = Stanton number
T = temperature
ρ = density
θ = momentum thickness
τ = shear stress
ν = kinematic viscosity

Subscripts

aw = adiabatic wall
e = edge condition
i = incompressible
N = nose
w = wall condition
0 = smooth wall

Introduction

THE computation of turbulent boundary-layer flow over re-entry bodies is important in the assessment of their performance. Surface roughness greatly influences the

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*Senior Aerodynamics Engineer, Dept. 81-11. Member AIAA.

boundary-layer flow, and its effects must be included in the design of re-entry bodies. An excellent review of the available methods for computing turbulent boundary layers on rough walls is given by Mills and Courtney.[1] These methods are broadly classified into integral methods and finite difference methods. Integral methods are simple, reliable, and fast if the associated empiricism is tolerable. Finite difference methods provide better modeling of the physical phenomena but are complex and time consuming. The integral methods are often preferred for design purposes, and finite difference methods are used to check the integral methods. In this Note the roughness effects are modeled by using appropriate correlations in the integral method and verified by the finite difference method. Both integral and finite difference methods have been used to predict the hypersonic, turbulent boundary-layer flow over a spherically blunted cone with rough walls. The predictions are evaluated by comparisons with experimental data obtained in wind tunnels. Comparisons for heat flux only have been made for lack of experimental data on skin friction. A brief description of the correlations used and the finite difference method are presented below, followed by the results and conclusions.

Analysis

Integral Method

Of the many integral methods available, the method of Hecht et al.[2] has been chosen, for it has been extensively used as a tool for the design of re-entry vehicles. The smooth-wall values of skin friction and heat transfer are corrected herein by the following correlations to include the roughness effects. The skin-friction relation of Goddard[3] correlates the skin-friction ratio of rough wall over smooth wall in terms of the roughness Reynolds number Re_k, and is given by

$$C_f/C_{f0} = 1.0 \qquad \text{for } Re_k < 10$$

$$= 0.88 \log(Re_k/10) + 1.0 \qquad \text{for } Re_k > 10$$

where

$$Re_k = k\sqrt{(\tau_w/\rho_w)}/\nu_w$$

This correlation is valid for adiabatic walls and is extended to nonadiabatic walls by Young's formula,[4] which relates the skin friction in compressible flow to that in the corresponding incompressible flow and is of the form

$$C_f/C_{fi} = 0.365\ T_e/T_{aw} + 0.635\ T_e/T_w$$

The heat-transfer relation of Owen and Thomson[5] has been chosen to correct the smooth-wall heat flux. This relation can be written in the form

$$\frac{St}{St_0} = \frac{C_f}{C_{f0}} Pr^{0.4} \Big/ \left(\frac{1+\sqrt{C_f/2}}{St_\delta} \right)$$

where the sublayer Stanton number St_δ is obtained by a correlation of the form $1/St_\delta = 0.7\ Re_k{}^{0.45}\ Pr^{0.8}$. The constant 0.7 has been chosen as it fits the recent data of Voisinet,[6] who verified all the preceding correlations using wind tunnel test data.

Although the stagnation-point flow is laminar, rough walls in experiments have shown a higher heat-transfer rate at the stagnation point than have smooth walls. Powars[7] developed a correlation to account for the roughness augmentation of stagnation-point heating. This correlation has been used in this Note to correct the laminar stagnation-point heating for rough walls.

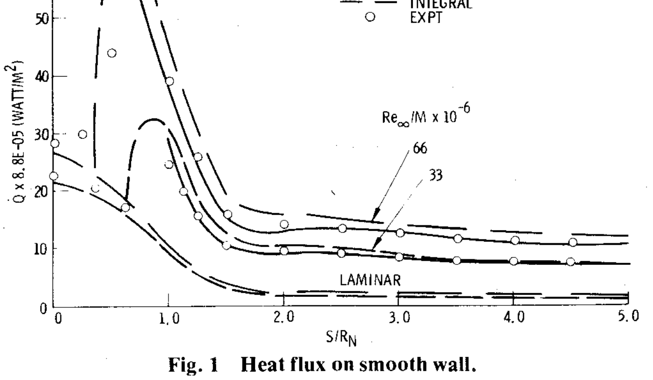

Fig. 1 Heat flux on smooth wall.

Finite Difference Method

The finite difference method, used was basically that of Bradshaw et al.[8] This method was chosen for its advantages, e.g., small computational times and a good turbulence model. The Reynolds stress terms are not modeled in any conventional way, but are retained as dependent variables, and transport equations are developed for them, thus closing the turbulent boundary-layer equations. This formulation makes the system of equations hyperbolic. This system then is solved by the method of characteristics. The initial conditions are the velocity and shear stress profiles on a starting line in the turbulent flow. If these profiles are not available, they are generated from given values of θ, Re_θ, and C_f using the Coles velocity profile family and an empirical mixing-length distribution. The computations are not continued down to the wall, but terminate one mesh point away from the wall, typically of sublayer thickness. The wall boundary condition is applied at this mesh point through "log law" used iteratively. The additive constant in the log law is modified to simulate Nikuradse's sand-roughness data.[9] The compressible log laws used for velocity and temperature are too complex to be given here. Details of the solution procedure can be found in Refs. 8 and 10.

Edge Conditions

The boundary-layer formulation requires specification of the edge conditions. The flow outside the boundary layer is inviscid and not isentropic because of the curved bow shock. A very fast and accurate method[11] has been used to solve the hypersonic inviscid flow over re-entry bodies and generate the inviscid pressure distribution. The entropy variation along the boundary-layer edge is determined by an approximate mass flow balance, and the edge properties are evaluated at this entropy and the local pressure.

The computation of the edge conditions associated with the body shape, the flow conditions, and the output of standard boundary-layer quantities constitutes a major portion of the code. The code was structured to accept any type of boundary-layer computation procedure, be it an integral or finite difference method. This concept was necessary for making consistent evaluations of different methods of boundary-layer computations.

Results

The integral method and the finite difference method previously described were used to predict the hypersonic turbulent boundary-layer flow over re-entry bodies with smooth and rough walls. A typical re-entry-body shape, a spherically blunted cone 5 nose radii in length with an 8-deg cone half-angle, was considered. The same body was tested in the wind tunnel[12] at a freestream Mach number of 5 and unit Reynolds numbers of 33 and 66×10^5/m. The roughness elements were brazed to the body surface. The predictions of

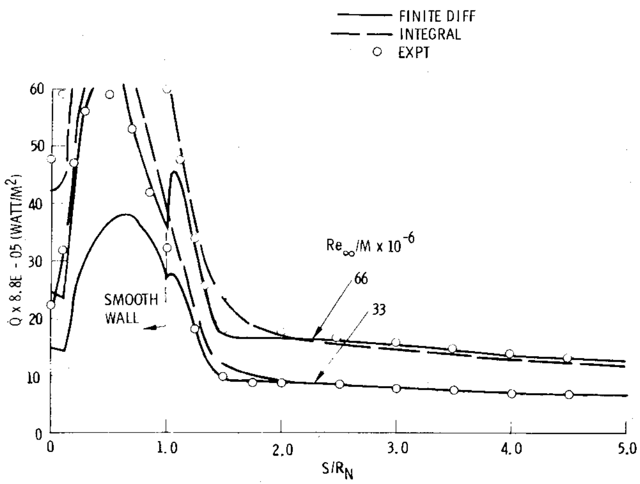

Fig. 2 Heat flux on a 0.076-mm rough wall.

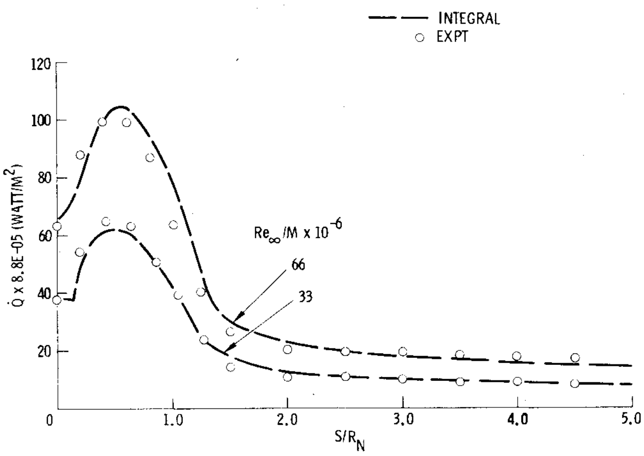

Fig. 3 Heat flux on a 0.254-mm rough wall.

wall heat flux only were compared with the test data for lack of data on skin friction.

Figure 1 shows the comparison of the predictions with the test data for smooth-wall heat flux at Reynolds numbers of 33 and 66×10^5/m. The predicted laminar value is also shown for reference. The finite difference solution starts around $S/R_N = 1.0$, since it requires a starting line in fully turbulent flow. Hence the turbulent smooth-wall solution from the stagnation point onward is supplied by the integral solution until the momentum thickness Reynolds number is around 600, where the finite difference solution is started. Both the integral and finite difference predictions agree well with the test data at the lower Reynolds number on the aft cone. But at the higher Reynolds number only the finite difference prediction agrees with the test data, whereas the integral method overpredicts by about 7% on the aft cone. The integral predictions are poor near the fore cone, close to the sphere-cone tangent point, where the pressure gradient is severe.

The heat flux on a 0.076-mm (3-mil) rough wall is shown for the two Reynolds numbers in Fig. 2. The roughness effects are predicted well by both the integral and finite difference methods at the lower Reynolds number. At the higher Reynolds number, the integral method underpredicts the heat flux by about 8% on the cone. Again it should be noted that the finite difference solution with roughness started around $S/R_N = 1.0$. The turbulent smooth-wall solution by the integral method, used from the stagnation point to $S/R_N = 1.0$, is shown for completeness and indicates the amount of stagnation-point heating augmentation due to roughness. The heat flux on a 0.254-mm (10-mil) rough wall is shown in Fig. 3 for the two Reynolds numbers. Only the integral prediction is shown, since the finite difference solution became unstable at roughness heights greater than 0.127 mm (5 mil). Again the integral method predicted the rough-wall heat flux well at the lower Reynolds number, but underpredicted by 11% at the high Reynolds number.

The instability of the finite difference method at higher roughness is probably due to a strong mismatch of the C_f in the input and the one in the starting profile. The Coles velocity profile used to generate the starting profile in the code does not have roughness effects. Possible solutions are finding an acceptable C_f by trial and error and modifying the mesh near the wall.

Average computer times on the UNIVAC 1100 were about 30 s for the integral solution and 45 s for the finite difference solution.

Conclusions

This study indicates the deficiencies of the integral method in the regions of strong pressure gradients and high Reynolds numbers. Under these conditions the finite difference predictions are better. Roughness effects are adequately modeled by the use of appropriate correlations in the integral method, for design purposes. The present finite difference method has been found to have limitations in regard to the size of roughness elements. The results indicate that the integral method can be used with confidence in the aerodynamic analysis of re-entry bodies.

Acknowledgments

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